# WAVES FROM AN UNDERGROUND EXPLOSION

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The problem of the propagation of a spherical detonation wave in water-saturated soil was solved in [1, 2] by using a model of a liquid porous multicomponent medium with bulk viscosity. Experiments show that soils which are not water saturated are solid porous multicomponent media having a viscosity, nonlinear bulk compression limit diagrams, and irreversible deformations. Taking account of these properties, and using the model in [2], we have solved the problem of the propagation of a spherical detonation wave from an underground explosion. The solution was obtained by computer, using the finite difference method [3]. The basic wave parameters were determined at various distances from the site of the explosion. The values obtained are in good agreement with experiment. Models of soils as viscous media which take account of the dependence of deformations on the rate of loading were proposed in [4-7] also. In [8] a model was proposed corresponding to a liquid multicomponent medium with a variable viscosity.

1. In accordance with the model in [2], we consider soil as a solid three-component medium containing free pore space filled with air, water, and mineral grains. We denote by $\alpha_1$, $\alpha_2$, and $\alpha_3$ the volumes of free pore space, water, and the solid component per unit volume of the medium; $\rho_{20}$ and $\rho_{30}$ are the densities of the liquid and the material of the solid component; $c_{20}$ and $c_{30}$ are the sound speeds in these materials. All quantities correspond to atmospheric pressure $p = p_0$, and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

The volume strain $\varepsilon$ of the medium includes the strain $\varepsilon_1$ of the free pore space arising as a result of the repacking of the solid and liquid particles under compression of the medium, and the strains $\varepsilon_2$ and $\varepsilon_3$ of the materials of the liquid and solid components: $\varepsilon = \alpha_1\varepsilon_1 + \alpha_2\varepsilon_2 + \alpha_3\varepsilon_3$.

The strain under compression is taken negative. The density of the medium at the initial pressure $p = p_0$ is

$$\rho_0 = \alpha_1\rho_{10} + \alpha_2\rho_{20} + \alpha_3\rho_{30},$$

where $\rho_{10}$ is the density of air.

The equations of static volume compression of the components as $\dot{p} \to 0$ and $\dot{\varepsilon} \to 0$ are approximated by the Tait equations.

For the free pore space

$$p - p_0 = f_S(\varepsilon_1) = \frac{\rho_0 c_S^2}{\gamma_S}\left[(\varepsilon_1 + 1)^{-\gamma_S} - 1\right].$$

For the remaining components

$$p - p_0 = \frac{\rho_{i0} c_{i0}^2}{\gamma_i}\left[(\varepsilon_i + 1)^{-\gamma_i} - 1\right], \quad i = 2; 3.$$

The compressibility of the first component — the free pore space — is substantially smaller than that of the air contained in it. The values of $\gamma_S$ and $\rho_0 c_S^2$ in the equation of compressibility are determined by the condition of repacking of the solid and liquid particles during deformation. They depend on the stiffness properties of the skeleton of the soil, and are found from experiment for each kind of soil separately. The equation of the static volume compression of the medium as $\dot{p} \to 0$ and $\dot{\varepsilon} \to 0$, in conformity with the equations of volume compressibility of the components, has the form

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$$\frac{V}{V_0} = \varepsilon + 1 = \alpha_1\left[\frac{\gamma_S(p-p_0)}{\rho_0 c_S^2} + 1\right]^{-\frac{1}{\gamma_S}} + \sum_{i=2}^{3}\alpha_i\left[\frac{\gamma_i(p-p_0)}{\rho_{i0}c_{i0}^2} + 1\right]^{-\frac{1}{\gamma_i}}.$$

The equations of dynamic volume compression of the materials of the liquid and solid components as $\dot{p} \to \infty$ and $\dot{\varepsilon} \to \infty$ are assumed the same as under static stress. They do not depend on the strain rate. The equation of dynamic volume compression of the free pore space is taken in the form $p - p_0 = f_S(\varepsilon_1) + k\varepsilon_1$, where $k < 0$.

Denoting by $k_S$ the static, and by $k_D$ the dynamic bulk modulus of the free pore space, we obtain $k = k_D - k_S$, and $k_S = -\rho_0 c_S^2$ as $p \to p_0$ and $\varepsilon_1 \to 0$.

As the soil is deformed, the description of its state shifts from the dynamic toward the static volume compression diagram. The rate of this shift is determined by the magnitude of the bulk viscosity $\eta$.

Experiments show that under small loads the deformation of the soil is due mainly to the decrease of the volume of the free pore space. Under loads of the order $(100-1000) \times 10^5$ $N/m^2$ the magnitude of the strain $\varepsilon_1$ of the free pore space approaches $\alpha_1$, and further increase of the strain of the medium under increased loading occurs mainly as a result of the compression of the material of the solid component and water. A similar character of the strain is taken into account in the model under consideration.

Under the assumptions made, the equation of volume compression of the medium has the form

$$\dot{\varepsilon} = \frac{\dot{V}}{V_0} = \varphi(p, V)\dot{p} - \frac{\alpha_1\lambda(p, V)}{\eta}\psi(p, V), \tag{1.1}$$

where

$$\varphi(p, V) = \alpha_1\left(\frac{df_D}{d\varepsilon_1}\right)^{-1} - \sum_{i=2}^{3}\frac{\alpha_i}{\rho_{i0}c_{i0}^2}\left[\frac{\gamma_i(p-p_0)}{\rho_{i0}c_{i0}^2} + 1\right]^{-\frac{1+\gamma_i}{\gamma_i}},$$

$$\lambda(p, V) = \frac{df_S}{d\varepsilon_1}\left(\frac{df_D}{d\varepsilon_1}\right)^{-1}, \quad \psi(p, V) = p - p_0 - f_S(\varepsilon_1),$$

$$f_D(\varepsilon_1) = f_S(\varepsilon_1) + k\varepsilon_1, \quad f_S(\varepsilon_1) = \frac{\rho_0 c_S^2}{\gamma_S}\left[(\varepsilon_1+1)^{-\gamma_S} - 1\right],$$

$$\varepsilon_1 = \frac{1}{\alpha_1}\left\{\frac{V}{V_0} - \sum_{i=2}^{3}\left[\frac{\gamma_i(p-p_0)}{\rho_{i0}c_{i0}^2} + 1\right]^{-\frac{1}{\gamma_i}}\right\} - 1, \quad \varepsilon = \frac{V - V_0}{V_0}.$$

It is assumed that the equation for the unloading of the materials of the liquid and solid components is the same as the equations for their compression. The equation of static unloading of the free pore space under maximum strains $|\varepsilon_1| > |\varepsilon_{1m}|$ agrees with the equation of compression; for smaller values of the maximum strain the unloading line is assumed parallel to the tangent to the compression curve at the point $\varepsilon_1 = \varepsilon_{1m}$.

Under these assumptions the equation for the volumetric unloading of the medium has the form (1.1), where

$$\varphi(p, V) = \alpha_1(k - \rho_0 c_R^2)^{-1} - \sum_{i=2}^{3}\frac{\alpha_i}{\rho_{i0}c_{i0}^2}\left[\frac{\gamma_i(p-p_0)}{\rho_{i0}c_{i0}^2} + 1\right]^{-\frac{1+\gamma_i}{\gamma_i}},$$

$$\lambda(p, V) = k(k - \rho_0 c_R^2)^{-1}, \quad \psi(p, V) = p - p_0 - f_R(\varepsilon_1),$$

$$f_R(\varepsilon_1) = \rho_0 c_R^2\left\{\varepsilon_1 - \left[\frac{\gamma_S(p_m - p_0)}{\rho_0 c_S^2} + 1\right]^{-\frac{1}{\gamma_S}} - \frac{p_m - p_0}{\rho_0 c_R^2} + 1\right\},$$

$$\rho_0 c_R^2 = \rho_0 c_S^2(\varepsilon_{1m} + 1)^{-\gamma_S - 1}, \quad 1 + \varepsilon_{1m} = \left[\frac{\gamma_S(p_m - p_0)}{\rho_0 c_S^2} + 1\right]^{-\frac{1}{\gamma_S}}.$$

Experiments show that the coefficient of lateral pressure $k_\tau = \sigma_\theta/\sigma_r$ in soil that is not water saturated increases with increasing pressure and approaches one as $p \to \infty$. For small loads the Mises-Schleicher plasticity condition is satisfied in the soil, and for larger loads the soil becomes similar to a liquid medium. Accordingly, the plasticity condition is taken in the form [9]

$$S_r = -\left(b_0 + \frac{k^*(p-p_0)}{1+\frac{k^*(p-p_0)}{p^*-p_0-b_0}}\right), \quad S_r = \sigma_r + (p - p_0). \tag{1.2}$$

Hence as $p \to p_0$, $p = p^*$, and $p \to \infty$, we obtain values of $k_\tau$ equal to $(2 - k^*)/(1 + k^*)2$, $(2 + k^*)/(1 + 2k^*)2$, and 1 respectively. As $p \to p_0$, $p = p^*$, and $p \to \infty$, $k_\tau = 0.38$, 0.56, and 1 respectively. Here it is assumed that $b_0 = 0$ and $p^* - p_0 = 2 \times 10^3 \times 10^5$ N/m$^2$.

The scheme of instantaneous wave detonation of the explosive charge was used in the solution. A two-term isentropic equation of state of the detonation products was assumed in the form [5, 10]

$$p = A\rho^{\gamma_a} + B\rho^{\gamma_b}. \tag{1.3}$$

For trotyl $\rho_n = 1600$ kg/m$^3$, $Q = 1000$ kcal/kg, $\gamma_\alpha = 3.12$, $\gamma_b = 1.25$, $A = 0.88$ (N/m$^2$)/(kg/m$^3$)$^{\gamma_\alpha}$, and $B = 0.62 \times 10^5$ (N/m$^2$)/(kg/m$^3$)$^{\gamma_b}$.

For spherical symmetry the equations of motion in Eulerian variables have the form

$$\begin{gathered}\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) - \frac{\partial \sigma_r}{\partial r} - \frac{2(\sigma_r - \sigma_\theta)}{r} = 0,\\ \frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2\rho u)}{\partial r} = 0, \quad p - p_0 = -\frac{1}{3}(\sigma_r + 2\sigma_\theta).\end{gathered} \tag{1.4}$$

Equations (1.1), (1.2), and (1.4) form a closed system. The closed system of equations of motion of the detonation products includes (1.3) and (1.4).

The initial conditions of the problem are: $u = 0$, $p = p_0$, $\rho = \rho_n$ for $0 \leqslant r \leqslant r_0$; $u = 0$, $\rho = \rho_0 = 1/V_0$, $p = p_0$ for $r_0 < r$.

On the boundary of the underground explosion cavity the stress $\sigma_r$ and the velocity $u$ are continuous.

The Lagrangian scheme used in the calculations with an artificial viscosity [3] enables one to take account of relations at the jump implicitly, without the removal of the singularity of the motion, if it ceases to be continuous. This leads to an artificial smearing out of the wave. A more accurate calculation enables one to narrow down the smeared-out region.

The values used for the soil characteristics correspond approximately to loess, in which the wave parameters have been measured for underground explosions of concentrated explosive charges under field conditions [11]. It was assumed that $\alpha_1 = 0.4$, $\alpha_2 = 0.2$, $\alpha_3 = 0.4$, $\rho_0 c_S^2 = 30 \times 10^5$ N/m$^2$, $k = 150 \times 10^5$ N/m$^2$, $\gamma_S = 6$, $\rho_{20} = 1000$ kg/m$^3$, $c_{20} = 1500$ m/sec, $\rho_{30} = 2650$ m/sec, $c_{30} = 5000$ m/sec, $\gamma_2 = 7$, $\gamma_3 = 5$, and $\varepsilon_{1m} = -0.15$. The radius of the charge $r_0 = 0.1$ m.

There are no experimental data on the bulk viscosity of the soil under consideration. Experiments in [12] show that the maximum and minimum possible values in clay and loamy soils are approximately 100 and 5000 N·sec/m$^2$. Calculations were performed for $\eta = 100$, 1000, and 5000 N·sec/m$^2$. The effect of the irreversibility of the deformations on the wave parameters was estimated by taking account of the differences between the compression and unloading limit diagrams of the medium, and without taking account of these differences, when the compression and unloading diagrams coincide for all pressures.

2. Let us consider the results of the calculation. Figure 1 shows the maximum radial stress $\sigma_r$ as a function of the dimensionless distance $R^\circ = r/r_0$ for wave propagation in various media. Lines 1 and 2 refer to the soil under consideration for $\eta = 5000$ and 100

$N \cdot sec/m^2$ respectively and for coincidence of the loading and unloading diagrams; 3 and 4 are for the soil under consideration, taking account of the differences in loading and unloading for $\eta = 5000$ and $1000\ N \cdot sec/m^2$; 5 and 6, taken from [4], correspond to water-saturated soil with different air contents for $\alpha_1 = 0.04$, $\alpha_3 = 0.6$ and $\alpha_1 = 0.01$, $\alpha_3 = 0.6$; 7 is for water. The calculations in this work were also performed for the scheme of instant detonation of the explosive charge. For water and water-saturated soil the graphs correspond to the pressure.

A comparison shows that the stress in the soil under consideration, which is not water saturated, decreases with distance much more rapidly than in water-saturated soils which are less compressible, and in water. This agrees with the experimental data. Increasing $\eta$ and taking account of the irreversibility of the deformations lead to stronger damping of the maximum stresses. An increase in $\eta$ by a factor of 50 changes the value of $\sigma_r$ by 30-40% for stresses of the order of $10 \times 10^5\ N/m^2$, and taking account of the irreversibility of the deformations changes it by 20-30%.

Curves 1 and 2 of Fig. 2 show the experimental dependence of the maximum radial ($\sigma_r$) and tangential ($\sigma_\theta$) stresses on the distance $R^0$ in loessial soil [11]. The masses of the concentrated explosive charges in the experiments were 0.2, 1.6, and 25 kg. In the last case $r_0 = 0.154$ m. Curves 3 and 4 correspond to calculations performed for $\sigma_r$ and $\sigma_\theta$ with $\eta = 1000\ N \cdot sec/m^2$, and 5 and 6 are for calculations of $\sigma_r$ and $\sigma_\theta$ with $\eta = 5000\ N \cdot sec/m^2$. Curves 3 and 5 coincide with curves 4 and 1 of Fig. 1. The calculated stress components are close to the experimental values. The best agreement is achieved for $\eta = 1000\ N \cdot sec/m^2$, when the differences between the compression and unloading diagrams are taken into account. Apparently still better agreement between the calculated and experimental values can be obtained by taking $\eta = 2000$-$2500\ N \cdot sec/m^2$.

Figure 3 shows the dependence of the maximum particle speed u on distance in the soil under consideration, in water-saturated soils, and in water. The notation is the same as in Fig. 1. The particle speed and the stress in soil that is not water saturated are smaller than in water-saturated soil and in water. The relative difference of the values of the speed, however, is smaller than that of the stress. Increasing the viscosity and taking account of the irreversibility of the deformations lead to a decrease of the particle speed. Curve 8 in Fig. 3 corresponds to the maximum values of the particle speed in experiments in loessial soil. The calculated values of the maximum particle speed and the maximum stresses are close to the experimental values.

Figure 4 shows the time dependence of the stress $\sigma_r$ during the passage of a wave at various distances from the site of the explosion for $\eta = 5000\ N \cdot sec/m^2$. Curves 1-3 correspond to distances $R^0 = 5.07$, 7.07, and 10.27 respectively. The differences between the compression and unloading diagrams were taken into account in the calculation. Curves 4-6 refer to $R^0 = 5.47$, 9.07, and 12.7 respectively when the compression and unloading diagrams were identical.

It follows from Fig. 4 that the detonation wave is smeared out as it propagates, and is transformed from a shock to a continuous compression wave. The time for the stress to rise to a maximum increases with distance in approximately the same way as in experiments [11]. A comparison of the curves shows that the wave profile is weakly dependent on taking account of the differences between the compression and unloading diagrams.

The wave may be smeared out by an artificial viscosity. An estimate performed by the method in [13] shows that the thickness of the smeared out region calculated with $\sigma_r = (100$-$120) \times 10^5\ N/m^2$ is greater than that produced by an artificial viscosity; i.e., it is related to the properties of the medium. Small oscillations of $\sigma_r(t)$ during the decrease of the stress do not follow from the model of the medium. This is a result of the method of calculation used on the computer. No additional smoothing of the curves was performed.

Figure 5 shows the change of state in particles of the medium during the passage of a wave. Curves 1 and 2 correspond to limit diagrams of dynamic and static compression of the medium; 3 and 4 show p as a function of V at distances $R^0 = 5.47$ and 9.07 for $\eta = 100\ N \cdot sec/m^2$; 5 and 6 are for the same distances, but for $\eta = 5000\ N \cdot sec/m^2$. In both cases the compression and unloading diagrams were assumed identical. Curves 7-9 correspond respectively to distances 5.07, 7.07 and 10.27 for $\eta = 5000\ N \cdot sec/m^2$, taking account of the differences between the dynamic and static compression diagrams.

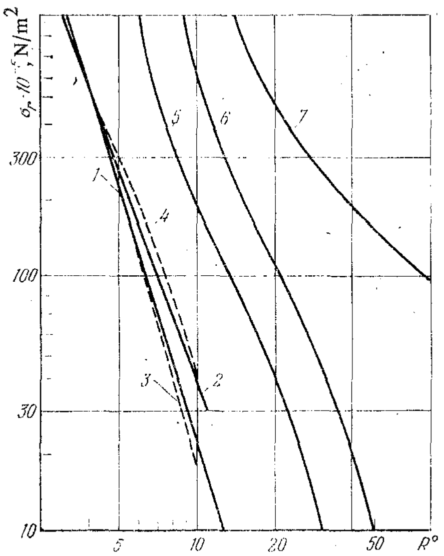


Fig. 1

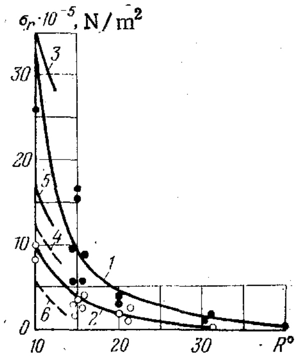


Fig. 2

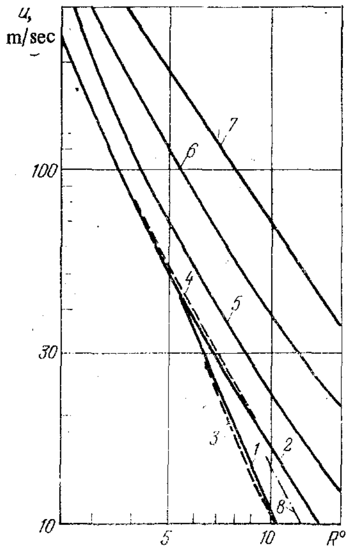


Fig. 3

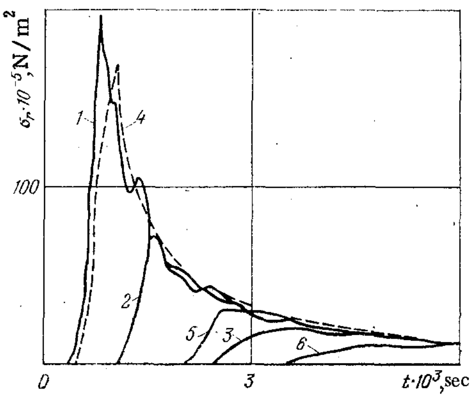


Fig. 4

At the distances considered the wave is already smeared out, the pressure is increasing without a jump, and therefore the parts of the p(V) curves corresponding to an increase in pressure lie between the limit compression diagrams. For smaller values of $\eta$ they are considerably closer to the static diagram. During part of the unloading of the medium these curves intersect the static diagram. At small distances the minimum volume is reached at the maximum pressure, and at larger distances while the pressure is decreasing. Experiments in which simultaneous measurements of stresses and strains were performed during the passage of a detonation wave [2] show that this is characteristic of soils.

The energy losses to particles of the medium during the passage of a wave are determined by the area of the figure in the (p, V) plane bounded by the p(V) curve corresponding to the compression and unloading of these particles. A comparison of the curves in Fig. 5 shows that an increase in $\eta$ leads to an increase in the area of this figure, i.e., to an increase in energy losses. The increase in energy losses is also related to the more rapid damping of the wave with distance for a larger value of $\eta$.

Figure 6 shows the time dependence of the dimensionless radius of the underground explosion cavity (gas chamber) $R_n = r_n/r_0$. Here $r_n$ is the dimensional radius of the cavity. Curves 1 and 2 correspond to $\eta = 5000$ and 1000 N·sec/cm². Calculations show that the $R_n(t)$ curves practically coincide whether or not the differences in the compression and unloading diagrams are taken into account.

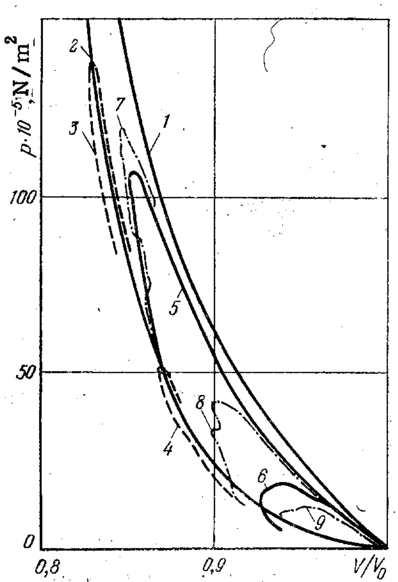

Fig. 5

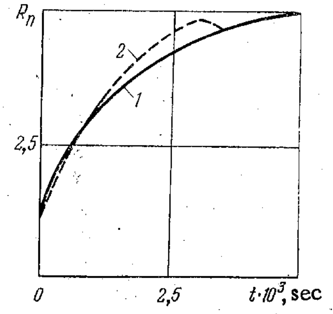

Fig. 6

TABLE 1

| $R_i^0 - R_{i+1}^0$ | D, m/sec | | |
|---|---|---|---|
| | η = 1000 N·sec/m² | η = 5000 N·sec/m² | Expt. |
| 3,07 — 4,27 | 600 | 450 | — |
| 4,27 — 5,07 | 400 | 350 | — |
| 5,07 — 7,07 | 300 | 150 | 280 |
| 7,07 — 10,27 | 160 | 150 | 170 |

The final dimensions of the cavities in underground explosions were measured in loessial soils. The average dimensionless radius of a cavity $R_n$ was 6.0. The curves in Fig. 6 show that the calculated value of the final radius of the cavity is nearly the same. The primary expansion of the gas cavity lasts about (6-8) × $10^{-3}$ sec. This time corresponds to the decrease of the maximum stress $\sigma_r$ in the wave to (2-4) × $10^5$ N/m².

Table 1 shows the average speeds D of propagation of the maximum stresses $\sigma_r$ over a succession of distance intervals $R^0$ from the site of the explosion. The calculated values of the speed were obtained for two bulk viscosities, taking account of the difference of the compression and unloading diagrams; the experimental values are for explosions in loessial soils.

Thus, the problem of the propagation of a spherical detonation wave produced in an underground explosion has been solved by using a model of a solid multicomponent medium and taking account of the plastic properties and bulk viscosity for nonlinear limit compression diagrams. The following basic wave parameters were determined; the maximum stress, the particle speed, the speed of propagation of the maximum stress, the wave profiles at various distances from the site of the explosion, and the radius of the underground cavity. A comparison with results of experiments performed in soils shows good agreement of the values of all the basic wave parameters. The model used [2] takes account of the basic properties of soils which determine the laws of wave processes.

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# NATURAL WAVENUMBERS OF ACOUSTIC AND ELECTROMAGNETIC OSCILLATIONS IN THE VICINITY OF A CIRCULAR CASCADE WITH A CORE

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For eigenvalue problems in diffraction theory the square of the wavenumber is usually adopted as the characteristic ("natural") parameter [1]. Rigorous and approximate methods are fairly well known for determining the natural wavenumbers (natural frequencies) of inner problems, but only the long-wave or short-wave approximations are considered for the most part in outer problems. The author is aware of only a few papers in which the eigenvalue problem has been solved in a rigorous setting (see, e.g., [2, 3] and the bibliographies therein). In the present article we determine the natural wavenumbers of the outer problem of the diffraction of electromagnetic or acoustic waves by a plane circular cascade with a core (hub) in a rigorous setting, i.e., for arbitrary ratios of the cascade period to the wavelength.

Circular cascades are customarily used to model the impellers or rotors of centrifugal compressors and fans. The solution of the eigenvalue problem should be useful in analyzing the acoustic resonance effect occurring in certain operating regimes of these machines [2]. Structures analogous to circular cascades can be regarded as models of electrodynamic resonators, certain antennas, and waveguide devices. To predict their resonance properties it is also necessary to know the natural wave numbers of electromagnetic oscillations in the vicinity of similar open structures [1].

1. We consider a stationary plane circular blade cascade of diameter 2R formed by N thin radial blades (reflectors) attached to a circular core of radius r (Fig. 1). Let the function $\varphi(\rho, \theta)$ describe the wave amplitude of steady-state acoustic or electromagnetic oscillations in the exterior of the cascade ($\rho$, $\theta$ are polar coordinates with origin at the center of the cascade). The amplitude of the total field can be written in the form

$$\varphi = \sum_{l=0}^{N-1} \varphi_l,$$

where each component $\varphi_l$ of the total field satisfies:

the homogeneous Helmholtz equation

$$(\Delta + k^2)\varphi_l = 0,$$

where k is an arbitrary complex number;

the homogeneous Dirichlet ($\Omega = 0$) or Neumann ($\Omega = 1$) conditions

$$\varphi_l = 0 \quad \text{or} \quad \partial\varphi_l/\partial \mathbf{n} = 0$$

on the blades forming the cascade and on the surface of the core;

the generalized radiation condition [3]

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